

STEAM - NOZZLES:

A nozzle is a duct of smoothly varying cross-sectional area in which a steadily flowing fluid can be made to accelerate by a pressure drop along the duct. Steam nozzles are used in steam & gas turbines, rocket motors etc. ~~which~~ when a fluid is decelerated in a duct, causing rise in ~~temp~~ pressure and decrease in velocity, it is called as diffuser.

Applying S.F.E.E to a nozzle.

$$h_1 + \frac{C_1^2}{2} + z_1 g + q_v = h_2 + \frac{C_2^2}{2} + z_2 g + q_w$$

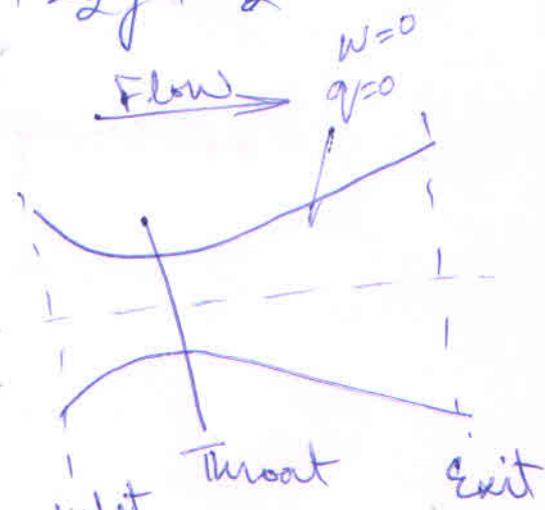
$$q_v = 0, w = 0, z_1 = z_2$$

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

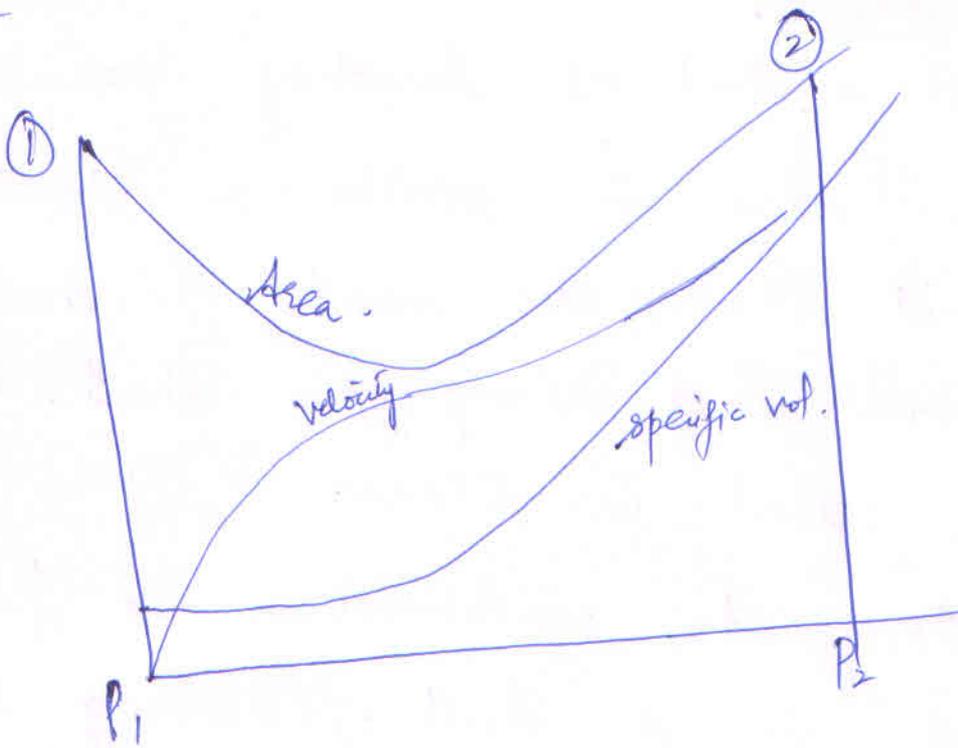
$$C_2 = \left[(h_1 - h_2) + \frac{C_1^2}{2} \right]^{1/2}$$

$$\dot{m} = \frac{C_2 A}{v} \quad \therefore \frac{A}{\dot{m}} = \frac{v}{C}$$

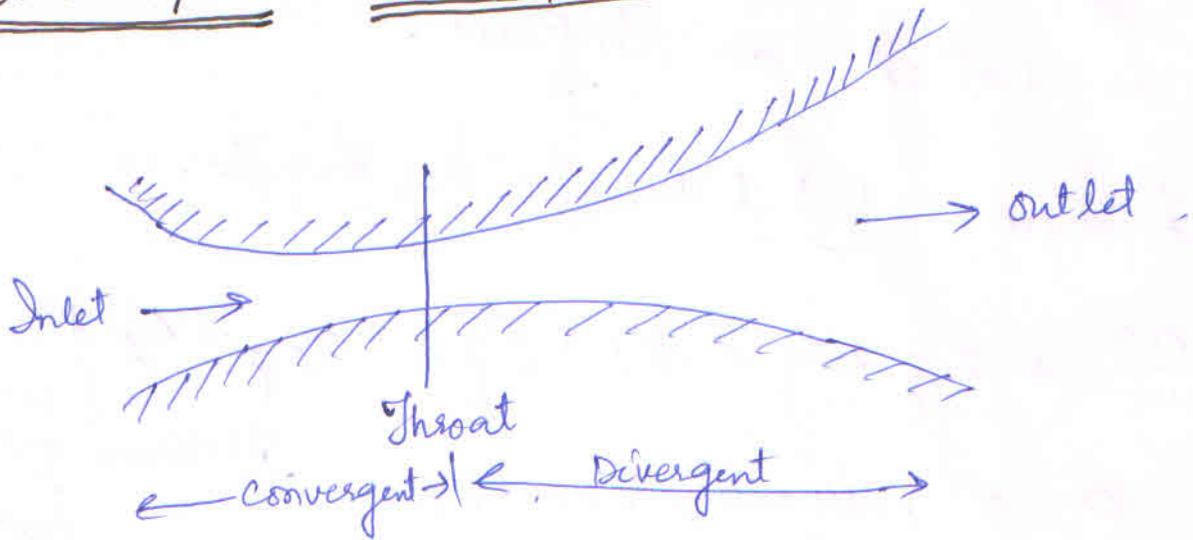
$$C_1 \ll C_2 \quad C_1 \approx 0$$



P-2



CONVERGENT - DIVERGENT NOZZLE :-

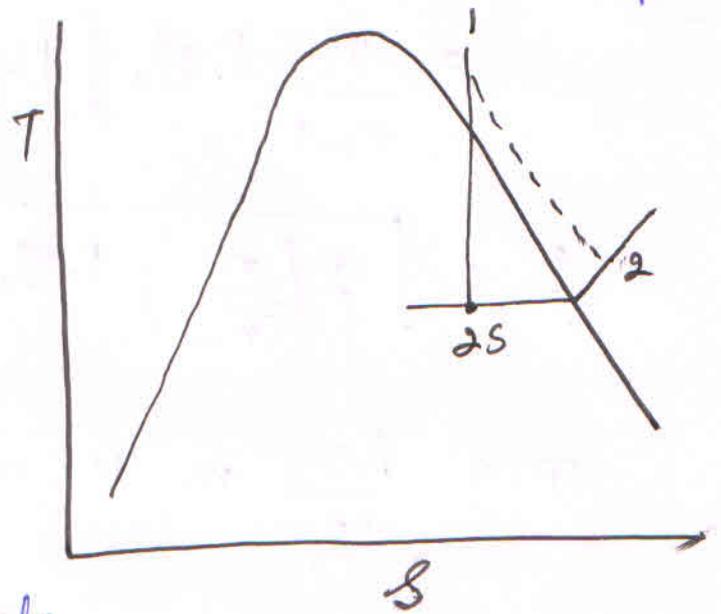


NOZZLE EFFICIENCY:

P-3

$$\eta = \frac{\text{Actual } \Delta h}{\text{Isentropic } \Delta h}$$

$$= \frac{h_1 - h_2}{h_1 - h_{2s}}$$



Discharge through nozzle.

Gain in k.E = Adiabatic heat drop
= work done during Rankine cycle.

$$\frac{C^2}{2} = \frac{\eta}{n-1} (P_1 v_1 - P_2 v_2)$$

$$= \frac{\eta}{n-1} P_1 v_1 \left(1 - \frac{P_2 v_2}{P_1 v_1} \right) \quad \text{--- (1)}$$

$$P_1 v_1^n = P_2 v_2^n = \frac{v_2}{v_1} = \left(\frac{P_1}{P_2} \right)^{\frac{1}{n}}$$

Putting $\frac{v_2}{v_1}$ in (1)

$$\frac{C^2}{2} = \frac{\eta}{n-1} P_1 v_1 \left[1 - \frac{P_2}{P_1} \left(\frac{P_1}{P_2} \right)^{\frac{1}{n}} \right]$$

P-4

$$= \frac{n}{n-1} P_1 V_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right]$$

$$C = \sqrt{2 \left(\frac{n}{n-1} \right) P_1 V_1 \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right\}}$$

$m = \frac{Ac}{V_2}$ m is mass discharged (kg/sec)

$$m = \frac{Ac}{V_1 \left(\frac{P_1}{P_2} \right)^{\frac{1}{n}}}$$

$$m = \frac{A}{V_1 \left(\frac{P_1}{P_2} \right)^{\frac{1}{n}}} \sqrt{2 \left(\frac{n}{n-1} \right) P_1 V_1 \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right]}$$

$$m = \frac{A}{V_1} \sqrt{2 \left(\frac{n}{n-1} \right) P_1 V_1 \left\{ \left(\frac{P_2}{P_1} \right)^{\frac{2}{n}} - \left(\frac{P_2}{P_1} \right)^{\frac{n+1}{n}} \right\}}$$

for max mass flow rate

$$\frac{P_2}{P_1} = \gamma$$

$$\frac{dm}{d\left(\frac{P_2}{P_1}\right)} = 0$$

$$\frac{2}{n} \left(\frac{P_2}{P_1} \right)^{\frac{2}{n}-1} - \left(\frac{n+1}{n} \right) \left(\frac{P_2}{P_1} \right)^{\frac{n+1}{n}-1} = 0$$

$$\left(\frac{P_2}{P_1} \right)^{\frac{2}{n}-1} = \frac{n+1}{2} \left(\frac{P_2}{P_1} \right)^{\frac{1}{n}}$$

$$\frac{P_2}{P_1} = \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}} \quad \text{Critical Pressure Ratio}$$

$$\frac{\text{Throat Press}}{\text{Inlet Press}} = \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}}$$

$n = 1.035$ Saturated Steam $\frac{P_2}{P_1} = 0.58$

$n = 1.3$ Superheated Steam $\frac{P_2}{P_1} = 0.546$

$$m_{\max} = A \sqrt{n \left(\frac{P_1}{V_1} \right) \left(\frac{2}{n+1} \right)^{\frac{n+1}{n-1}}}$$

m_{\max} depends on only n , P_1 & V_1 and is independent of V_2 & P_2 .

RELATION BETWEEN A, C and P in nozzle:- P-6

$$m = \frac{AC}{V}$$

$$\frac{dA}{A} + \frac{dc}{c} - \frac{dv}{V} = 0 \quad \text{--- (1)}$$

$$Pv^\gamma = k \quad \frac{dp}{P} + \gamma \frac{dv}{V} = 0$$

$$\frac{dv}{V} = -\frac{1}{\gamma} \frac{dp}{P} \quad \text{--- (2)}$$

$$Cdc = -Vdp$$

$$\frac{dc}{c} = -\frac{Vdp}{c^2} \quad \text{--- (3)}$$

Combining (1), (2) & (3)

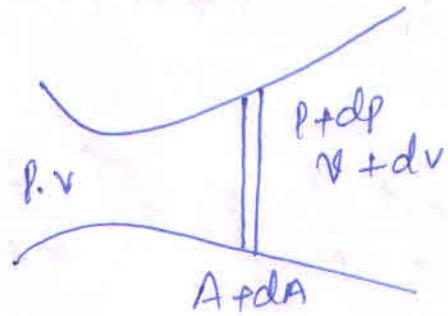
$$\frac{dA}{A} - \frac{Vdp}{c^2} + \frac{1}{\gamma} \frac{dp}{P} = 0$$

$$\frac{dA}{A} = \frac{Vdp}{c^2} - \frac{1}{\gamma} \frac{dp}{P}$$

$$= \frac{1}{\gamma} \left[\frac{Vdp}{c^2} \cdot \frac{\gamma P}{dp} - 1 \right] \frac{dp}{P}$$

$$= \frac{1}{\gamma} \frac{dp}{P} \left(\gamma \frac{Vdp}{c^2} - 1 \right)$$

$$\gamma Pv = c^2 = \gamma RT$$



$$\frac{dA}{A} = \frac{1}{\gamma} \frac{dP}{P} \left(\frac{C_s^2}{c^2} - 1 \right)$$

$$= \frac{1}{\gamma} \frac{dP}{P} \left(1 - \frac{M^2}{M^2} \right) \quad M = \frac{c}{C_s}$$

Acc. Flow

$M < 1$ $\frac{dA}{A}$ must be negative (Convergent)
as $dP < 0$

$M > 1$ $\frac{dA}{A}$ must be positive (Divergent)
as $dP < 0$

Decelerated Flow

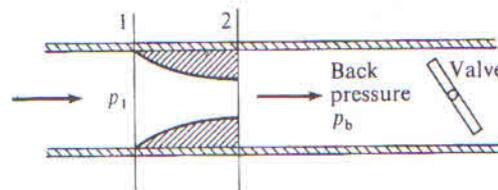
$M < 1$ $\frac{dA}{A}$ must be positive (Divergent)
 $dP > 0$

$M > 1$ $\frac{dA}{A}$ must be negative (Convergent)
 $dP > 0$

10.3 Maximum mass flow

Consider a convergent nozzle expanding into a space, the pressure of which can be varied, while the inlet pressure remains fixed. The nozzle is shown diagrammatically in Fig. 10.6. When the back pressure, p_b , is equal to p_1 , then no fluid can flow through the nozzle. As p_b is reduced the mass flow through the nozzle increases, since the enthalpy drops, and hence the velocity, increases. However, when the back pressure reaches the critical value, it is found that no further reduction in back pressure can affect the mass flow. When the back pressure is exactly equal to the critical pressure, p_c , then the velocity at exit is sonic and the mass flow through the nozzle is at a maximum. If the back pressure is reduced below the critical value then the mass flow remains at the maximum value, the exit pressure remains at p_c , and the fluid expands violently outside the nozzle down to the back pressure. It can be seen that the maximum mass flow through a convergent nozzle is obtained when the pressure ratio across the nozzle is the critical pressure ratio. Also, for a convergent-divergent nozzle, with sonic velocity at the throat, the cross-sectional area of the throat fixes the mass flow through the nozzle for fixed inlet conditions.

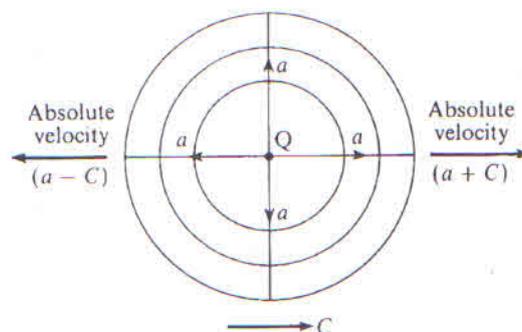
Fig. 10.6 Convergent nozzle with back-pressure variation



When a nozzle operates with the maximum mass flow it is said to be *choked*. A correctly designed convergent-divergent nozzle is always choked.

An attempt can be made to explain the phenomenon of choking, by considering the velocity of any small disturbance in the stream. Any small disturbance in the flow is propagated as small pressure waves travelling at the velocity of sound in the fluid in all directions from the centre of the disturbance. This is illustrated in Fig. 10.7; the pressure waves emanate from point Q at the velocity of sound relative to the fluid, a , while the fluid moves with a velocity, C . The absolute velocity of the pressure waves travelling back upstream is therefore given by $(a - C)$. Now when the fluid velocity is subsonic, then $C < a$,

Fig. 10.7 Propagation of a small disturbance in a flowing fluid



and the pressure waves can move back upstream; however, when the flow is sonic, or supersonic (i.e. $C = a$ or $C > a$), then the pressure waves cannot be transmitted back upstream. It follows from this reasoning that in a nozzle in which sonic velocity has been attained no alteration in the back pressure can be transmitted back upstream. For example, when air at 10 bar expands in a nozzle, the critical pressure can be shown to be 5.283 bar. When the back pressure of the nozzle is 4 bar, say, then the nozzle is choked and is passing the maximum mass flow. If the back pressure is reduced to 1 bar, say, the mass flow through the nozzle remains unchanged. Even if the air were allowed to expand into an evacuated space, the mass flow would be no greater than that through the nozzle when the back pressure is 5.283 bar.

Example 10.2

A fluid at 6.9 bar and 93 °C enters a convergent nozzle with negligible velocity, and expands isentropically into a space at 3.6 bar. Calculate the mass flow per square metre of exit area:

- (i) when the fluid is helium ($c_p = 5.19$ kJ/kg K);
- (ii) when the fluid is ethane ($c_p = 1.88$ kJ/kg K).

Assume that both helium and ethane are perfect gases, and take the respective molar masses as 4 kg/kmol and 30 kg/kmol.

Solution (i) It is necessary first to calculate the critical pressure in order to discover whether the nozzle is choked.

From equation (2.9), $R = \bar{R}/\bar{m}$, therefore for helium,

$$R = \frac{8314.5}{4} = 2079 \text{ N m/kg K}$$

Then from equation (2.22)

$$c_p = \frac{\gamma R}{(\gamma - 1)}$$

i.e. $\frac{\gamma - 1}{\gamma} = \frac{R}{c_p} = \frac{2079}{10^3 \times 5.19} = 0.4$

therefore

$$\gamma = \frac{1}{1 - 0.4} = 1.667$$

Then using equation (10.7)

$$\frac{p_c}{p_1} = \left(\frac{2}{\gamma + 1} \right)^{\gamma/(\gamma - 1)} = \left(\frac{2}{2.667} \right)^{1.667/0.667} = 0.487$$

i.e. $p_c = 0.487 \times 6.9$ bar

i.e. Critical pressure $p_c = 3.36$ bar

The actual back pressure is 3.6 bar, hence in this case the fluid does not reach the critical conditions and the nozzle is not choked.

Nozzles off the design Pressure Ratio:

→ UNDER EXPANSION: When the back pressure is below the design value, the nozzle is said to Underexpand. In Underexpansion the fluid expands to the design press. in the nozzle and expands violently and irreversibly down to the back press. on leaving the nozzle.

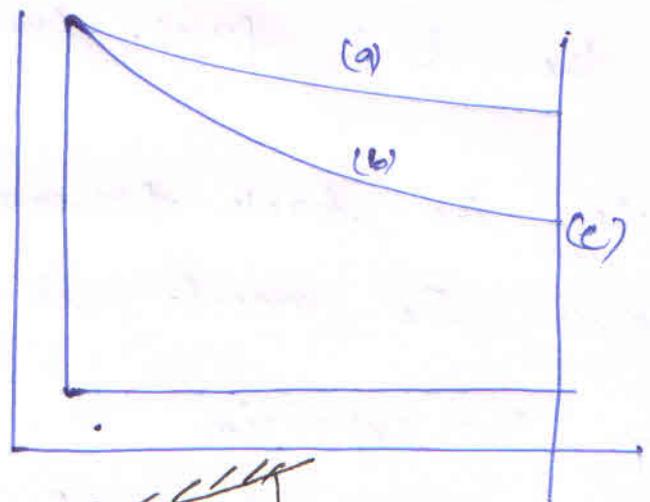
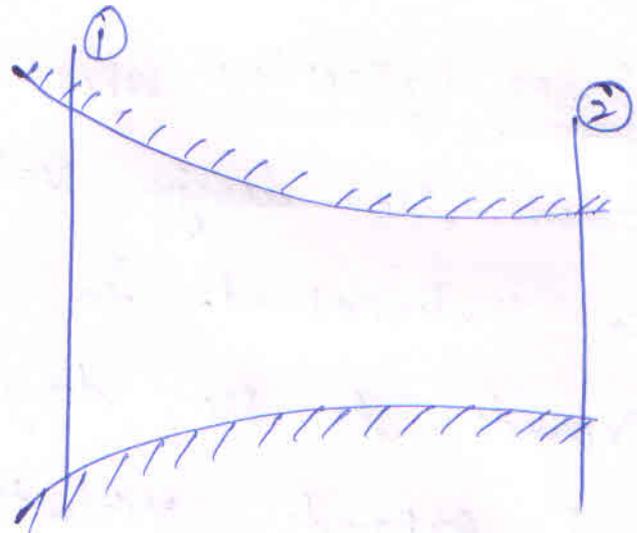
→ When the back pressure is above the design value the nozzle is said to Overexpand. In Overexpansion in a convergent nozzle the exit press is greater than the critical press. and the effect is to reduce the mass flow through nozzle. In Overexpansion in a C-D nozzle there is always an expansion followed by a recompression.

CONVERGENT NOZZLE:-

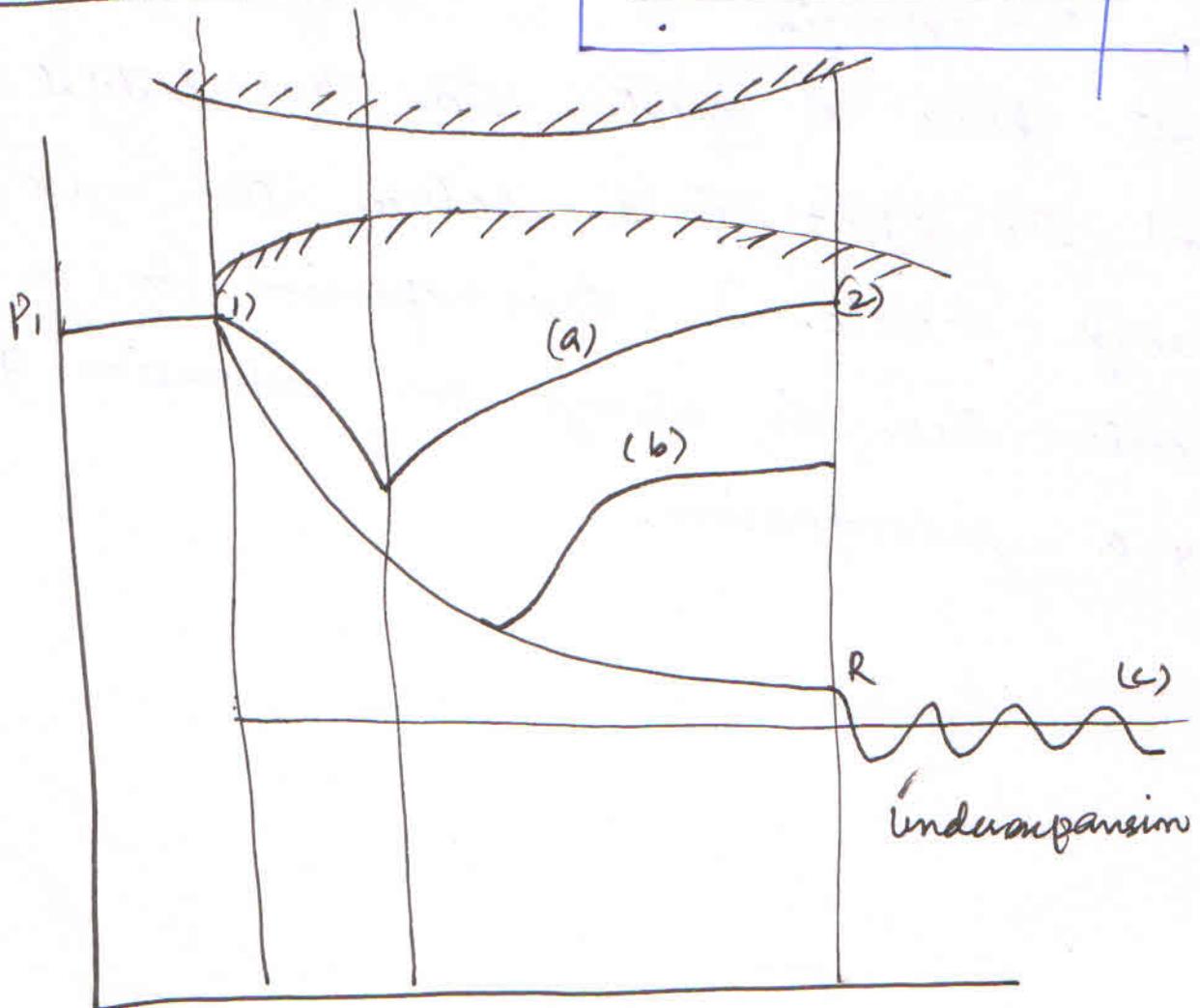
a = Over expansion

b = Critical expansion

c = Expansion Outside the nozzle.



CONVERGENT-DIVERGENT NOZZLE:-



Off the design pressure ratio

P. 8

→ Underexpansion: When the back pressure is below the design value, the nozzle is said to Underexpand. In Underexpansion the fluid expands the design press. in the nozzle and expands violently and irreversibly down to the back pressure on leaving the nozzle.

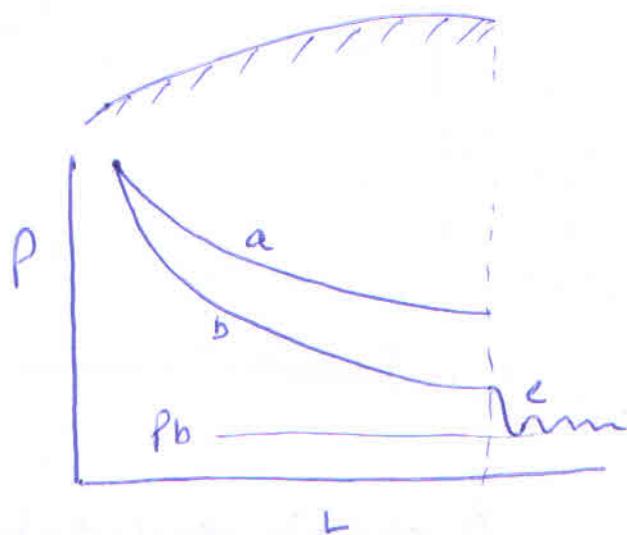


A - Expansion

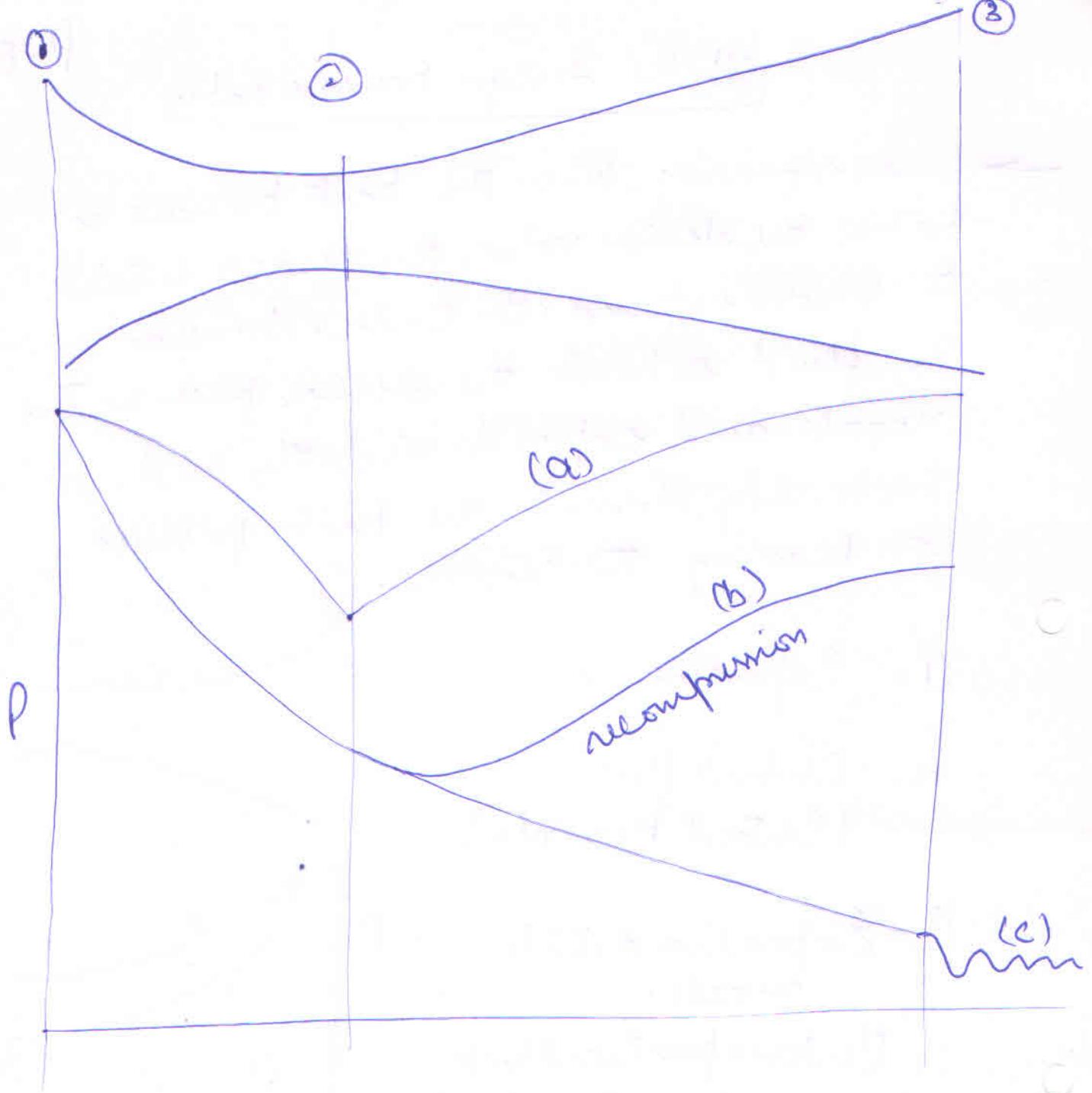
B - Choked flow
(Critical press. ratio)

C = Expansion outside nozzle.

Underexpansion shock waves formed violently



→ When the back press. is above the design value the nozzle is said to be expanded. In overexpansion, the nozzle exit press. is greater than the Crit. press. and the effect is to reduce the mass flow through nozzle. In overexpansion there is expansion followed by a compression



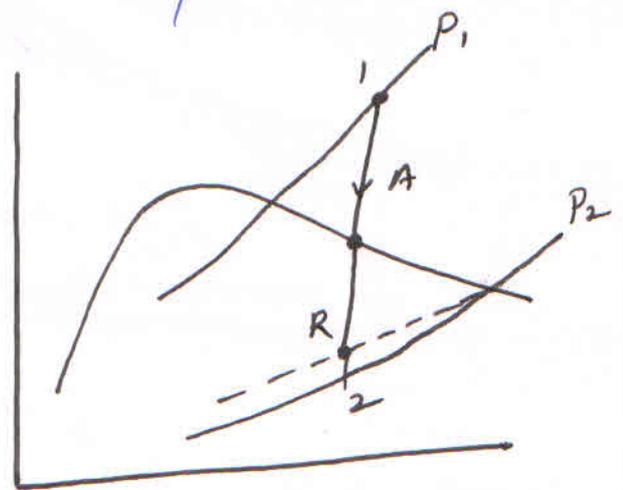
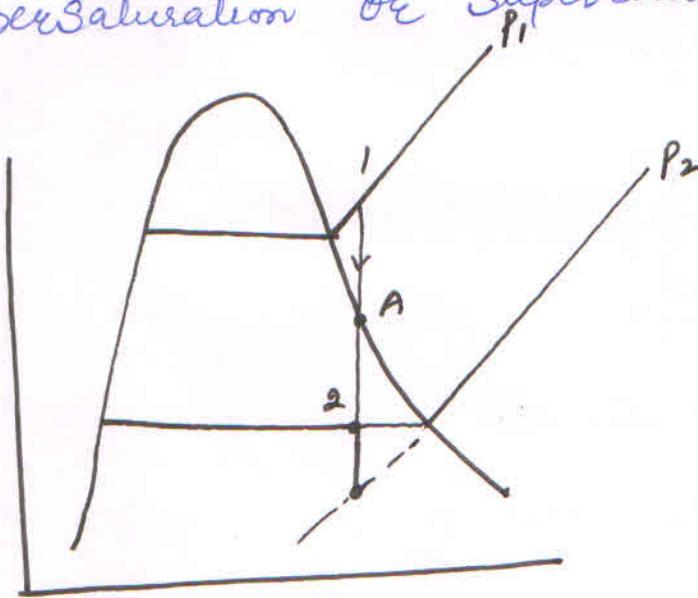
a and b are expansion

c is adiabatic expansion

When supersonic stream is decelerated a shock wave results hence recompression (b). It is irreversible recompression through shock wave. Both a & b are overexpansion.

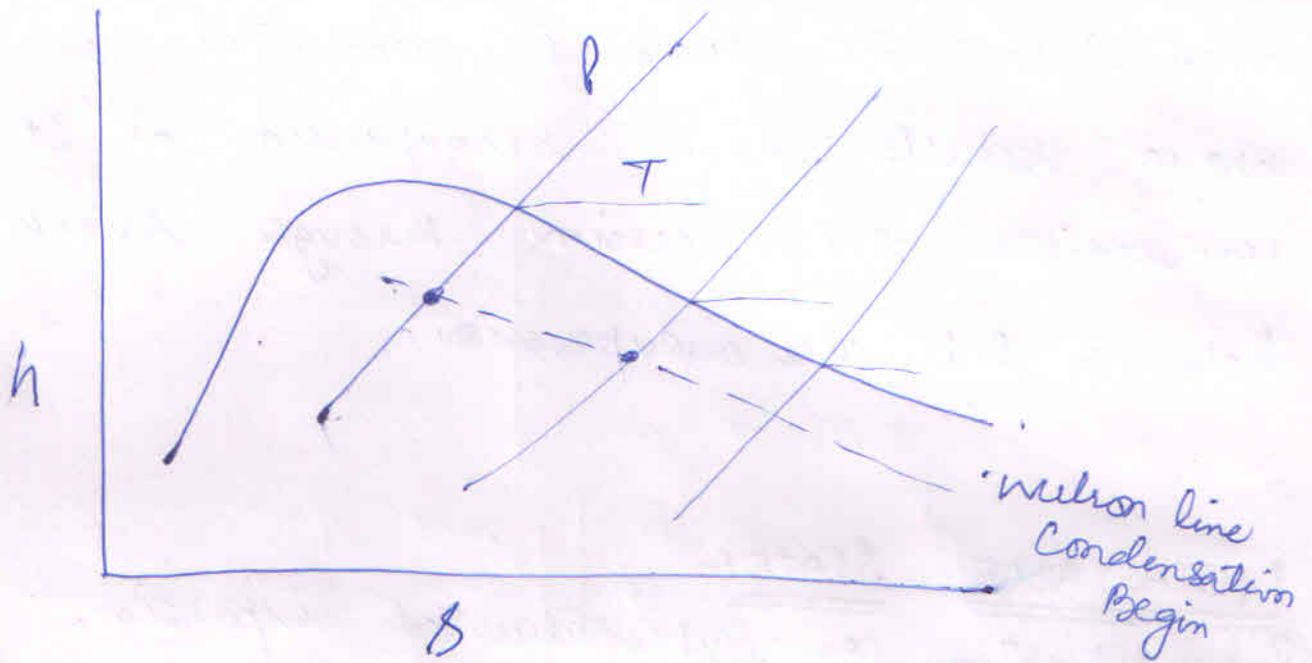
METASTABLE STATE:-

Supersaturation or supersaturated expansion.



Temperature of supersaturated vapour at P_2 is T_R . which is less sat. temp T_2 at P_2 -
 Vapour is said to be super cooled and
 the degree of supercooling is given by
 $(T_2 - T_R)$

$$\text{Degree of Saturation} = \frac{P_2}{\text{Sat Press at } T_R}$$



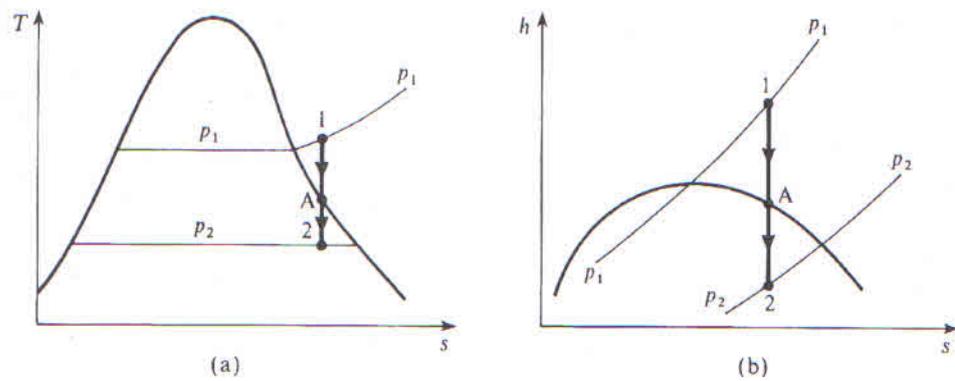
i.e. Throat area per kilogram per second = 0.20 m²

Supersaturation

When a superheated vapour expands isentropically, condensation within the vapour begins to form when the saturated vapour line is reached. As the expansion continues below this line into the wet region, then condensation

Nozzles and Jet Propulsion

Fig. 10.15 Superheated steam expanding into the wet region on (a) $T-s$ and (b) $h-s$ diagrams



proceeds gradually and the dryness fraction of the steam becomes progressively smaller. This is illustrated on $T-s$ and $h-s$ diagrams in Figs 10.15(a) and (b). Point A represents the point at which condensation within the vapour just begins.

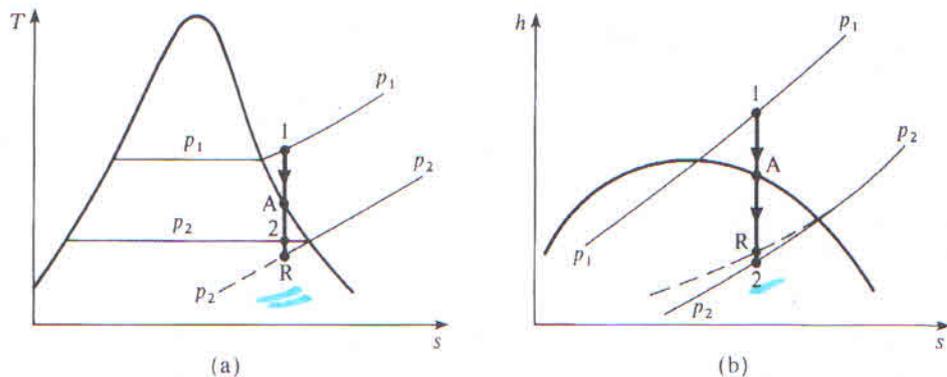
It is found that the expansion through a nozzle takes place so quickly that condensation within the vapour does not occur. The vapour expands as a superheated vapour until some point at which condensation occurs suddenly and irreversibly. The point at which condensation occurs may be within the nozzle or after the vapour leaves the nozzle.

Up to the point at which condensation occurs the state of the steam is not one of stable equilibrium, yet it is not one of unstable equilibrium, since a small disturbance will not cause condensation to commence. The steam in this condition is said to be in a metastable state; the introduction of a large object (e.g. a measuring instrument) will cause condensation to occur immediately.

Such an expansion is called a supersaturated expansion.

Assuming isentropic flow, as before, a supersaturated expansion in a nozzle is represented on a $T-s$ and an $h-s$ diagram in Figs 10.16(a) and (b) respectively. Line 1-2 on both diagrams represents the expansion with equilibrium throughout the expansion. Line 1-R represents supersaturated expansion. In supersaturated expansion the vapour expands as if the vapour line did not exist, so that line 1-R intersects the pressure line p_2 produced from the superheat region (shown chain-dotted). It can be seen from Fig. 10.16(a) that the temperature of the supersaturated vapour at p_2 is t_R , which is less than the saturation temperature t_2 , corresponding to p_2 . The vapour is said to be

Fig. 10.16 Supersaturated expansion of steam on (a) $T-s$ and (b) $h-s$ diagrams



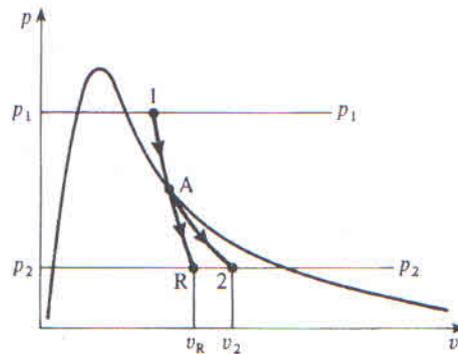
supercooled and the *degree of supercooling* is given by $(t_2 - t_R)$. Sometimes a *degree of supersaturation* is defined as the ratio of the actual pressure p_2 to the saturation pressure corresponding to the temperature t_R .

It can be seen from Fig. 10.16(b) that the enthalpy drop in supersaturated flow $(h_1 - h_R)$ is less than the enthalpy drop under equilibrium conditions. Since the velocity at exit, C_2 , is given by equation (10.4), $C_2 = \sqrt{2(h_1 - h_2)}$, it follows that the exit velocity for supersaturated flow is less than that for equilibrium flow. Nevertheless, the difference in the enthalpy drop is small, and since the square root of the enthalpy drop is used in equation (10.4), then the effect on the exit velocity is small.

If the approximations for isentropic flow are applied to the equilibrium expansion, then for the process illustrated in Figs 10.16(a) and (b), the expansion from 1 to A obeys the law $pv^{1.3} = \text{constant}$, and the expansion from A to 2 obeys the law $pv^{1.135} = \text{constant}$. The equilibrium expansion and the supersaturated expansion are shown on a $p-v$ diagram in Fig. 10.17, using the same symbols as in Fig. 10.16. It can be seen from Fig. 10.17 that the specific volume at exit with supersaturated flow, v_R , is considerably less than the specific volume at exit with equilibrium flow, v_2 . Now the mass flow through a given exit area, A_2 , is given by equation (1.11), i.e. for equilibrium flow

$$\dot{m} = \frac{A_2 C_2}{v_2}$$

Fig. 10.17 Equilibrium and supersaturated expansion processes on a $p-v$ diagram



and for supersaturated flow

$$\dot{m}_s = \frac{A_2 C_R}{v_R}$$

It has been pointed out that C_2 and C_R are very nearly equal; therefore, since $v_R < v_2$, it follows that the mass flow with supersaturated flow is greater than the mass flow with equilibrium flow. It was this fact, proved experimentally, that led to the discovery of the phenomenon of supersaturation.

Example 10.5

A convergent-divergent nozzle receives steam at 7 bar and 200°C and expands it isentropically into a space at 3 bar. Neglecting the inlet velocity, calculate the exit area required for a mass flow of 0.1 kg/s:

- (i) when the flow is in equilibrium throughout;
- (ii) when the flow is supersaturated with $pv^{1.3} = \text{constant}$.

STEAM Nozzles:

Q10.1 → Calculate the throat and exit areas of a nozzle to expand air at the rate of 4.5 kg/s from 8.3 bar, 327°C into a space at 1.38 bar. Neglect the inlet velocity and assume isentropic flow.

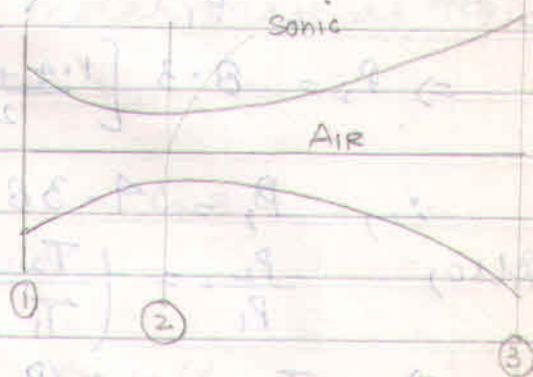
SOL

Given, $\dot{m} = 4.5 \text{ kg/s}$

$P_1 = 8.3 \text{ bar}$

$T_1 = 327^\circ\text{C} = 600 \text{ K}$

$P_3 = 1.38 \text{ bar}$



$\frac{C_p}{R} = 1.4$

Isentropic flow.

$$\Rightarrow \frac{P_3}{P_1} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \left(\frac{T_3}{T_1}\right) = \left(\frac{P_3}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \left| \gamma_{\text{air}} = 1.4 \right.$$

$$\Rightarrow T_3 = T_1 \left(\frac{P_3}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 600 \left(\frac{1.38}{8.3}\right)^{\frac{0.4}{1.4}}$$

$\therefore T_3 = 359.35 \text{ K}$

$$\begin{aligned} \therefore C_3 &= \sqrt{2(h_1 - h_3)} = \sqrt{2c_p(T_1 - T_3)} \\ &= \sqrt{2(1.005)(600 - 359.35) \times 10^3} \quad \left| C_p = 1.005 \text{ kJ/kg} \right. \\ &= 695.5 \text{ m/s} \end{aligned}$$

Also, $P_3 v_3 = R T_3 \quad \left| R_{\text{air}} = 287 \text{ kJ/kg} \right.$

$$\Rightarrow v_3 = \frac{R T_3}{P_3} = \frac{(287)(359.35)}{1.38 \times 10^5}$$

$$\therefore v_3 = \frac{0.76395148}{0.747344} \text{ m}^3/\text{kg}$$

$$\therefore, A_{exit} = \frac{\dot{m} v_3}{C_3} = \frac{(4.5) (0.747344)}{695.5} = 0.76395178$$

$$\therefore, A_{exit} = 4835.44 \text{ mm}^2$$

At throat, velocity is sonic

$$\therefore, C_2 = a = \sqrt{\gamma R T_2}; T_2 = ?$$

We have, $\frac{P_2}{P_1} = \left(\frac{\gamma+1}{2}\right)^{\gamma/\gamma-1}$

$$\Rightarrow P_2 = 8.3 \left[\frac{1.4+1}{2}\right]^{\frac{1.4}{0.4}}$$

$$\therefore, P_2 = 4.38474 \text{ bar}$$

Also, $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/\gamma-1}$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 600 \left[\frac{4.38474}{8.3}\right]^{\frac{0.4}{1.4}}$$

$$\therefore, T_2 = 500 \text{ K}$$

$$P_2 v_2 = R T_2$$

$$\Rightarrow v_2 = \frac{(287)(500)}{4.38474 \times 10^5} = 0.3273$$

$$\therefore, v_2 = 0.3273 \text{ m}^3/\text{kg}$$

$$C_2 = \sqrt{\gamma R T_2} = \sqrt{(1.4)(287)(500)} = 448.2187 \text{ m/s}$$

$$\therefore, A_2 = \frac{\dot{m} v_2}{C_2} = \frac{(4.5) (0.3273)}{448.2187}$$

$$\Rightarrow A_2 = 3285.72 \text{ mm}^2$$

$$\text{Coefficient of velocity} = \frac{\text{Actual velocity}}{\text{Isentropic velocity}}$$

$$= \frac{680}{853.41} = 0.7968 \approx 0.80$$

- Q10.5 → Steam enters a convergent-divergent nozzle at 11 bar, dry saturated at a rate of 0.75 kg/sec and expands isentropically to 2.7 bar. Neglecting the inlet velocity, and assuming the expansion follows a law, $pv^{1.35} = \text{constant}$, calculate;
- the area of the nozzle throat;
 - the area of the nozzle exit.

SOL

Given, $P_1 = 11 \text{ bar}$.

$P_2 = 2.7 \text{ bar}$.

$\dot{m} = 0.75 \text{ kg/sec}$

Now,

$h_1 = 2779.7 \text{ kJ/kg}$

$s_1 = 6.566 \text{ kJ/kg K}$

$s_2 = s_1 = s_{f2} + x_2 s_{fg2}$

$\Rightarrow 6.566 = 546.24 + x_2 (2173.7)$

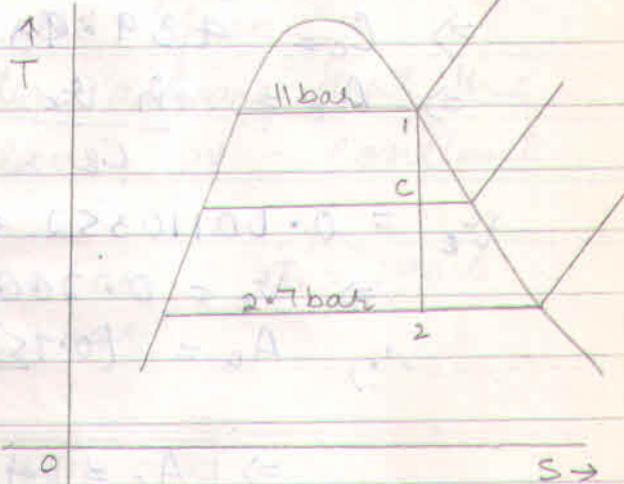
$6.566 = 1.034 + x_2 (5.392) \Rightarrow x_2 = 0.9147$

$\therefore h_2 = h_{f2} + x_2 h_{fg2} = 546.24 + 0.9147 (2173.7)$

$\Rightarrow h_2 = 2534.34 \text{ kJ/kg}$

$C_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(2779.7 - 2534.34) \times 10^3}$

$\therefore C_2 = 700.51 \text{ m/sec}$



$$\frac{P_2}{P_1} = \left(\frac{2}{n+1} \right)^{n/n-1}$$

$$P_2 = P_1 \left(\frac{2}{n+1} \right)^{n/n-1} = 11 \left[\frac{2}{2.135} \right]^{\frac{1.135}{0.135}}$$

$$\Rightarrow P_2 = 6.352 \text{ bar}$$

$$S_2 = S_1 = S_{f_2} + x_c S_{fg_c}$$

$$\Rightarrow 6.566 = 1.95312 + x_c (4.78552)$$

$$\Rightarrow x_c = 0.964$$

$$\therefore, h_c = 680.204 + x_c (2077.684)$$

$$\Rightarrow h_c = 2602.93 \text{ kJ/kg}$$

$$C_c = \sqrt{2(h_1 - h_c)}$$

$$= \sqrt{2(2779.7 - 2602.93) \times 10^3}$$

$$\Rightarrow C_c = 439.932 \text{ m/s}$$

$$\Rightarrow A_c = \frac{\dot{m} v_c}{C_c}$$

$$v_c = 0.00110352 + x_c (0.2989696 - 0.00110352)$$

$$\Rightarrow v_c = 0.288246$$

$$\therefore, A_c = \frac{(0.75)(0.288246)}{439.932}$$

$$\Rightarrow A_c = 491.4 \text{ mm}^2$$

$$v_2 = 0.001133 + x_2 (0.17739 - 0.001133)$$

$$\Rightarrow v_2 = 0.162355 \text{ m}^3/\text{kg}$$

$$\therefore, A_2 = \frac{\dot{m} v_2}{v_2} = \frac{(0.75)(0.162355)}{700.51}$$

$$\Rightarrow A_2 = 173.825 \text{ mm}^2$$

$$v_2 = 0.001070 + \eta_2 (0.66040 - 0.001070)$$

$$\Rightarrow v_2 = 0.611476$$

$$\therefore A_2 = \frac{\dot{m} v_2}{c_2} = \frac{(0.75)(0.611476)}{(700.51)}$$

$$\Rightarrow \underline{A_2 = 654.67 \text{ mm}^2}$$

Q10.6 → Steam enters a convergent-divergent nozzle at 11 bar

Steam at 20 bar and 240°C expands isentropically to a pressure of 3 bar in a convergent-divergent nozzle. Calculate the mass flow per unit exit area:

i) assuming equilibrium flow.

ii) assuming supersaturated flow.

For supersaturated flow assume that the process follows the law, $pv^{1.3} = \text{constant}$.

Sol.

We are given, $P_1 = 20 \text{ bar}$; $T_1 = 240^\circ\text{C}$

$P_2 = 3 \text{ bar}$

$$h_1 = 2074.42 \text{ kJ/kg}$$

$$s_1 = s_2 = 6.4897 \text{ kJ/kg K}$$

$$6.4897 = 1.672 + \eta (5.319)$$

$$\Rightarrow \eta = 0.90575$$

$$h_2 = 561.5 + \eta (2163.2) = 2520.82 \text{ kJ/kg}$$

$$v_2 = 0.001074 + \eta (0.60553 - 0.001074)$$

$$\Rightarrow v_2 = 0.54856 \text{ m}^3/\text{kg}$$

$$c_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(2074.42 - 2520.82) \times 10^3}$$

$$\Rightarrow c_2 = 840.952 \text{ m/s}$$

$$\Rightarrow \frac{\dot{m}}{A_2} = \frac{C_2}{v_2} = \frac{840.952}{0.54856}$$

$$\therefore \boxed{\frac{\dot{m}}{A_2} = 1533 \text{ kg/s}}$$

Supersaturated flow,

$$p_1 v_1^{1.3} = p_2 v_2^{1.3}$$

$$v_1 = 0.100322 \text{ m}^3/\text{kg}$$

$$v_2 = \left(\frac{p_1}{p_2}\right)^{1/1.3} \times v_1 = 0.46611 \text{ m}^3/\text{kg}$$

$$\Rightarrow v_2 = 0.46611 \text{ m}^3/\text{kg}$$

$$\frac{C_R^2}{2} = \frac{k}{k-1} (p_1 v_1 - p_2 v_2)$$

$$\Rightarrow C_R = \sqrt{2 \left(\frac{1.3}{0.3}\right) \left\{ 20 \times 10^5 (0.100322) - 3 \times 10^5 (0.46611) \right\}}$$

$$\Rightarrow C_R = 015.901 \text{ m/s}$$

$$\therefore \frac{\dot{m}}{A_2} = \frac{C_R}{v_2} = \frac{015.901}{0.46611}$$

$$\therefore \boxed{\frac{\dot{m}}{A_2} = 1750 \text{ kg/s}}$$

Q10.3 → Recalculate Problem 10.1, assuming a coefficient of discharge of 0.96 and a nozzle efficiency of 0.92

SOL

Given, Coeff of discharge = 0.96

$$\eta_{\text{nozzle}} = 0.92$$

Students are advised to go through the lecture notes along with the following books

- 1. Engineering Thermodynamics by P K NAG**
- 2. Applied Thermodynamics for Engineering Technologists by EASTOP & McCONKEY**

In case of any typographic mistake, error or any difficulty, students are advised to call me on 9906763424,7006161837, hanief@nitsri.net

students can call me for arranging video lectures

Students must complete this module within 5 days i.e before

(20th May).

Three unsolved numerical have been solved from EASTOP (Prob. 1 5 &6). Students are advised to attempt other problems from EASTOP and PK Nag.